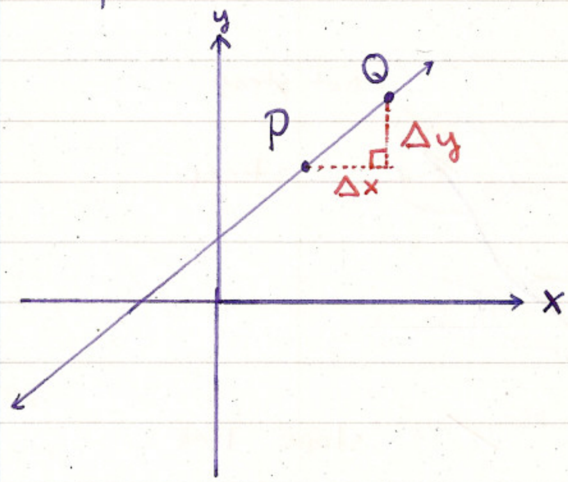
A Rough Overview of Calculus

a. Differential Calculus

In its simplest form, this branch of calculus is concerned with finding "slopes" of curves / functions / graphs.

Slope of a line

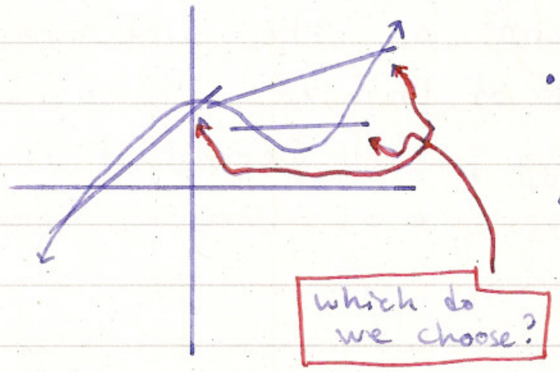


- Take two points, $P = (x_1, y_1)$ and $Q = (x_2, y_2)$. Slope is just: (Δ means "change in")

$$\frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

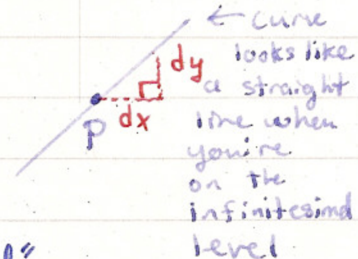
- Easy!
- Slope is the same anywhere on the line

Slope of a curve

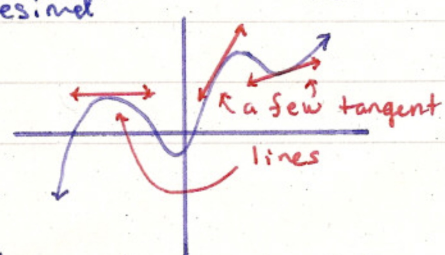


- Harder. What do we mean by slope?
- Whatever we mean, the "slope" will be different on different parts of the curve.

Resolution:

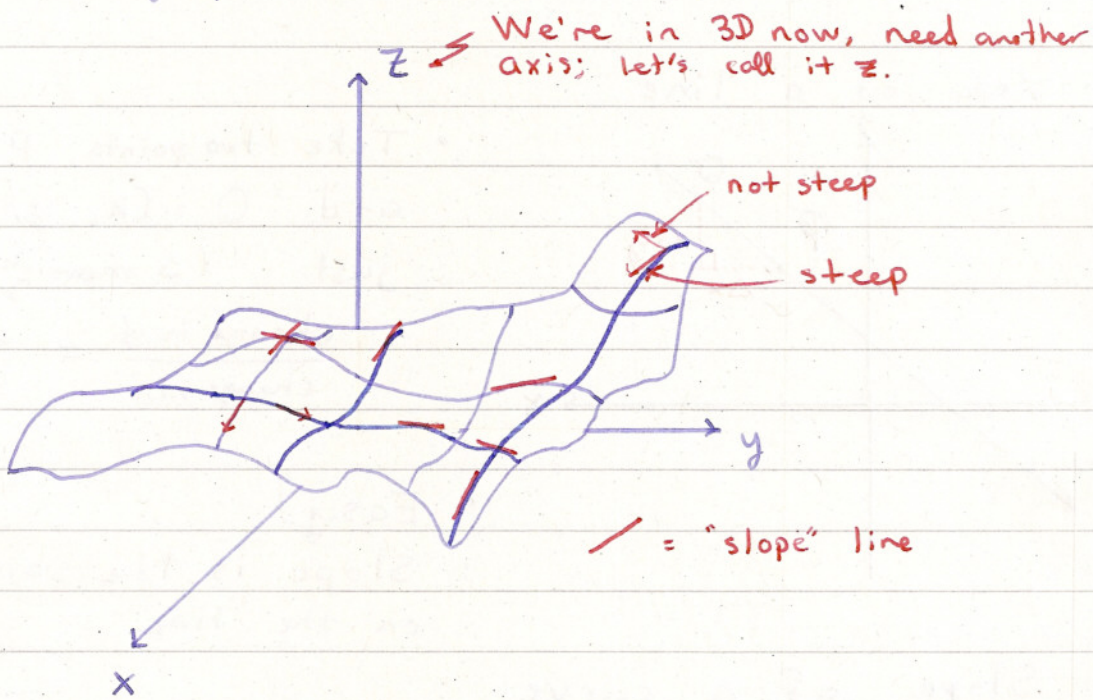


For a point P, the derivative (= "slope") of the curve is the "infintesimal" change in y resulting from an "infinitesimal" change in x, divided by this "infinetesimal" change in x: write $\frac{dy}{dx}$



In summary, we decide that a sensible definition of the "slope" of a curve is the slope of the tangent line at each point a (continued)

Can generalise to more "interesting" mathematical objects, e.g. the three-dimensional equivalent of a graph: a "landscape"

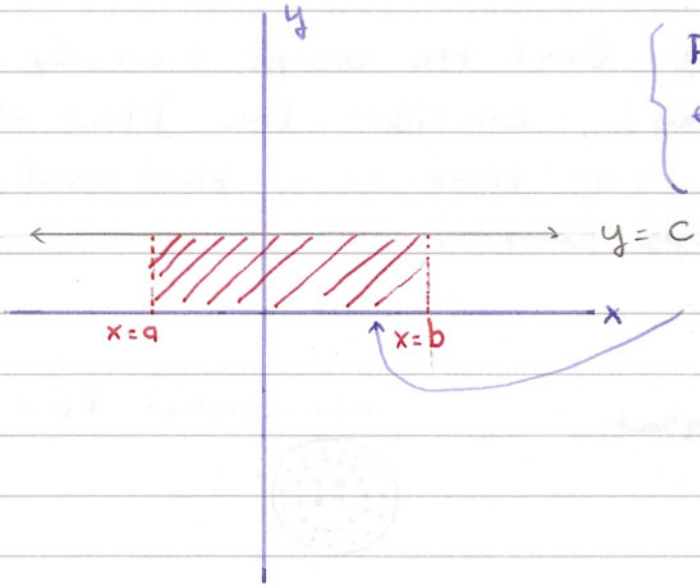


- Slope obviously depends on the direction you're facing (think of this as a landscape of rolling hills; not craggy peaks! Derivatives of "spiky" things are not as nice as those of "smooth" things.

b. Integral Calculus

In its simplest form, this branch of calculus is concerned with finding "the area under a curve" (or under a function, or graph, depending on what terminology you use; they're all similar)

For a flat line (with equation: y = a fixed number, c)



• What is this area? Well, its obviously the area of a rectangle, and in this case:

$$\text{Area} = \text{"length"} \times \text{"width"}$$
$$= (b-a) \times c$$

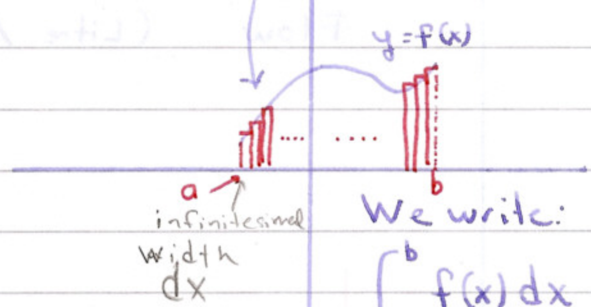
• Similarly, the area under any straight line (from $x=a$ to $x=b$) can be found. The area will be that of a trapezoid. If you like, try this for yourself (you can probably find the trapezoid area formula on the web). You should get, for a general line $y = mx + c$,

$$\text{Area} = \tfrac{1}{2}mb^2 + cb - \tfrac{1}{2}ma^2 - ca.$$

What about for curves?

For curves, it's harder. Basically, we divide the area we want to find into "strips" of infinitesimal width, then add up the (infinitely many) strips.

on a small scale, curve looks like a line, which looks a bit like a flat line.



We write:

$$\int_a^b f(x)\,dx$$

for the area ...

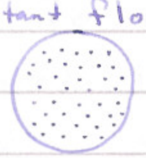
Again, we can generalise to more interesting mathematical objects

- A not so interesting example: The volume of the "tent" underneath the "landscape" in the earlier differentiation example

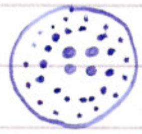
- More interesting (and the example I actually gave on Wednesday), consider the flow of water in a pipe (or blood in a blood vessel, if you want a biology example)

cross section:

constant flow throughout



non-constant flow



(Bigger dots = higher rate of flow)

In the first example, the flow rate of fluid (per area) is constant, so the flow rate for the entire section of tube is just (Area) x (Flow rate per area). However, in the second example, we must break up the cross-section into small patches where the flow per area is constant (small = infinitesimal here). Then

$$\text{Flow (Litre/sec)} = \int_{\text{pipe cross section}} \underbrace{F(x,y)}_{\left(\frac{L/sec}{m^2}\right)} \, \underbrace{dA}_{(m^2)}$$

(adding up all patches)

area of small patch around (x,y)

(Flow rate per area at point $P=(x,y)$, putting cross section on an x-y grid)